An UL Multi-user FDD Massive MIMO scheme with efficient lossless Feedback Compression

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Abstract Quantized feedback overhead compression is considered in this paper. In fact, FDD multi-user Massive MIMO OFDM can be considered as the promising technique for providing unprecedented spectral efficiency. However, due to the large number of antennas, it requires excessive feedback overhead, and this constitute a real handicap. In this paper, an improvement of the conventional Givens Rotation (GR) lossless feedback overhead compression based on an adaptation of the Lempel-Ziv-Welch (LZW) algorithm is proposed. Simulation results show that whatever the system configuration, the proposed technique named GR&LZW outperforms the GR one and achieves a compression rate which exceeds 90%.

Index Terms— Massive MIMO, CSI, FDD, Feedback overhead compression, OFDM, loss less compression, multi-user.

◆

1 INTRODUCTION

Massive MIMO is a promising technique for providing unprecedented spectral efficiency [1]. Several powerful pres-processing and post-processing techniques are proposed for the Beamforming and the elimination of the potential multi-user interference (MUI) [2], [3], [4], [5], [6], [7]. As in conventional MIMO systems, OFDM is also considered in Massive MIMO in order to greatly reduce the inter-symbol interference (ISI) [8].

However, in FDD multi-user Massive MIMO, uplink (UL) and downlink (DL) channel state information (CSI) are mandatory at each transceiver side [9]. Due to the fact that the number of antenna is very important, FDD Massive MIMO OFDM systems requires an important feedback overhead. Consequently, limited feedback is now an important research topic and substantial efforts are being made in this direction. Therefore, several solutions are proposed.

We can mention codebook-based techniques where channel distribution is taken into account. Instead of the CSI overhead, an index of the selected CSI in the designed codebook is fed back. However, as the codebook is restricted to have fixed cardinality, the channel quantization error increases with the growth of antenna ports in massive MIMO [10].

There are other techniques based on the fact that in massive MIMO systems, channels matrices tend to be sparse due to the limited local scatters of base stations (BS) [11],[12]. To reduce the feedback amount, the estimated frequency CSI is first converted into a transform domain. As the channel matrix is supposed to be sparse, only the significant channel coefficients are considered [10], [13], [14], [15], [16]. The feedback overhead reduction rate of this kind of technique depends on the optimisation of the threshold from which some of the channel coefficients in the transform domain are considered to be insignificant.

There are even recent studies that propose the use of Deep Learning. But for the moment, this only concerns low rate massive MIMO systems [17], [18].

In this paper we focus on lossless feedback compression in UL Massive MIMO ODFM systems specially in Givens Rotation (GR) approach [19] which is adopted in the IEEE 802.11 standard [20]. This GR lossless compression technique can achieve a compression rate up to 75%, but this rate decreases with the growth of antennas number.

Based on this observation, we propose an enhancement of the GR based on an adaptation of the Lempel-Ziv-Welch (LZW) algorithm. The LZW is a universal lossless data compression algorithm which is simple to implement and has the potential for very high throughput in hardware implementations [21].

The remainder of this paper is organized as follows. In section 2, we present the considered UL Massive MIMO FDD system model. Next, in section 3, the principle of Givens Rotation feedback compression approach is explained before the presentation of the proposed improvement. Finally, simulation results that show the interest of the proposed feedback overhead compression are presented in section 4, and conclusions follow in section 5.

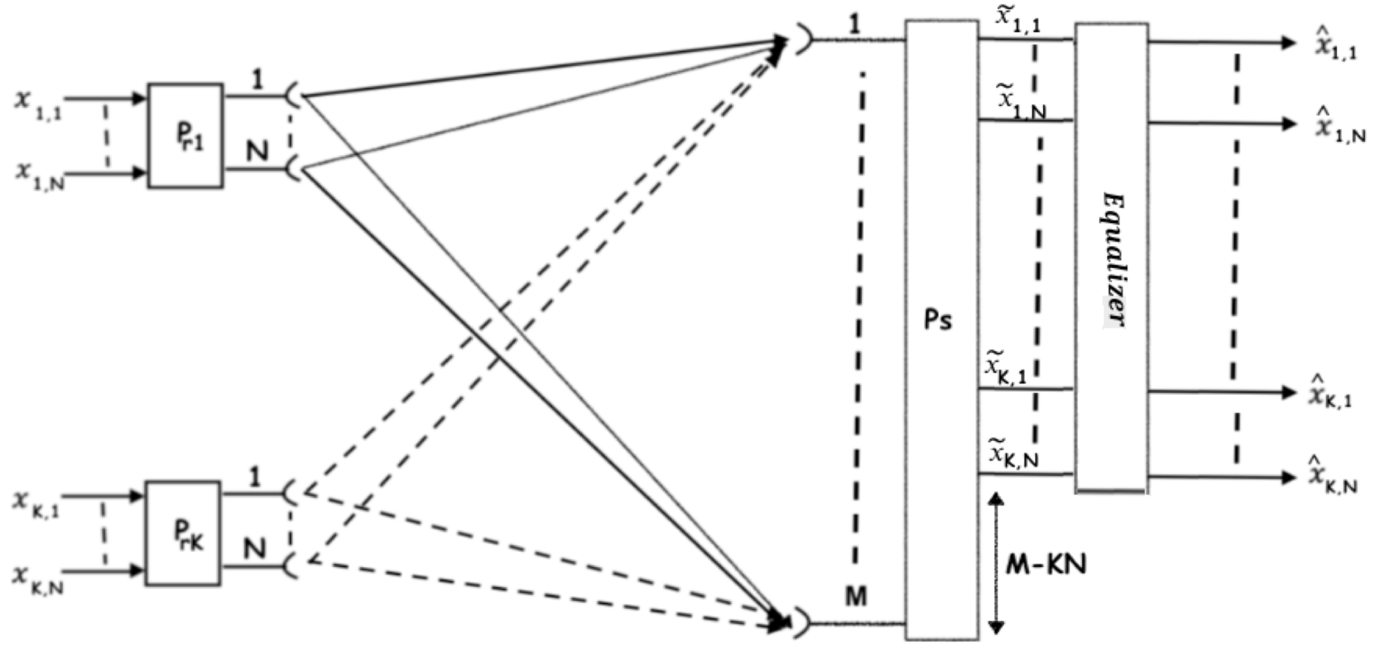

Fig. 1. Multi-user multi-antenna Uplink FDD massive MIMO scheme with Pres and post-processing.

2 THE UL MULTI-USER MASSIVE MIMO MODEL WITH PRES-CODING AND POST-CODING

2.1 The system model

We consider, as shown in Fig.1, an UL multi-user Massive MIMO OFDM system, where the BS supports K UEs. The BS is equipped with M antennas while each UE has N antennas, where M is supposed to be greater than or equal to $N \times K$. The system is assumed to operate in FDD mode. Note that, in order to avoid cluttering the Fig.1, only one OFDM subcarrier is considered.

In Fig.1, the vector $x_u = [x_{u,1}, \dots, x_{u,N}]$ represents the transmitted data of the u$^{\text{th}}$ UE, where $u = 1, \dots, K$. A pres-processing is done at each UE side, using the pres-coding matrices P_{ru} of size $N \times N$, where $u = 1, \dots, K$. Next, all UEs send their pres-processed data in the same bandwidth at the same time.

Let the Frequency Channel Response (FCR), on one OFDM subcarrier, between the u$^{\text{th}}$ UE and the BS be expressed as:

$$H_u = \begin{bmatrix} H_{1,1} & H_{1,2} & \cdots & H_{1,N_{UE}} \\ H_{2,1} & H_{2,2} & \cdots & H_{2,N_{UE}} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N_{BS},1} & H_{N_{BS},2} & \cdots & H_{N_{BS},N_{UE}} \end{bmatrix} \qquad (1)$$

The frequency domain received signal, at the BS for the considered OFDM subcarrier can then be expressed as:

$$y = \sum_{u=1}^{K} H_u P_{ru} x_u + n \qquad (2)$$

Wheren is a length-M additive white Gaussian noise with zero mean and a covariance matrix given by $\sigma^2 I_M$. A post-processing with the post-coding matrix P_s of size $M \times NK$ is performed at the BS in order to eliminate the MUI.After that, the equalization stage gives the detected data $\hat{x} = [\hat{x}_{1,1}, \dots, \hat{x}_{1,N}, \dots, \hat{x}_{u,1}, \dots, \hat{x}_{u,N}, \dots, \hat{x}_{K,1}, \dots, \hat{x}_{K,N}]$ on the considered OFDM subcarrier. The process described in this section is the same for all OFDM subcarriers. To facilitate paper reading, in the following, we will consider a single OFDM subcarrier.

2.2 The SVD based Beamforming

Consider the singular value decomposition (SVD) of the FCR matrix h_u:

$$H_u = U_u D_u V_u \qquad (3)$$

Where $D_u \in \mathbb{C}^{N_{BS} \times N_{UE}}$ is diagonal and $U_u \in \mathbb{C}^{N_{BS} \times N_{BS}}$ and $V_u \in \mathbb{C}^{N_{UE} \times N_{UE}}$ are unitary matrices. In SVD based beamforming, the transmit signal is precoded using the matrix $P_{ru} = V^T$, where†stands for the transpose-conjugation.

In this context, the Eq.2 becomes:

$$y = \sum_{u=1}^{K} U_u D_u x_u + n \qquad (4)$$

2.3 The ZF based MUI cancellation

Based on Eq.4, it is not possible to perform separately the post-processing which allows the transmitted data of each UE to be detected at the BS side. As shown in Fig.1, one postcoder matrix P_s of size $M \times NK$ is used to separate signals from the K different UEs.

As proposed in [2], let's look at the structure of matrices U_u and D_u. Since M is larger than N, let us assume that the rank of H_u is N. Then, the Eq.4 can be expressed as:

$$y = \sum_{u=1}^{K} [U_{ul} U_{ur}] \begin{bmatrix} D_{ul} \\ 0 \end{bmatrix} x_u + n \qquad (5)$$

Where U_{ul} contains the N left eigen-vectors corresponding to the signal subspace, U_{ur} contains the $M - N$ right eigen-vectors corresponding to the null subspace and D_{ul} contains the N non zero eigenvalues of $H_u (H_u)^T$. Then Eq.4 becomes now:

$$y = \sum_{u=1}^{K} U_{ul} D_{ul} x_u + n \qquad (6)$$

This sum of matrices products can be reformulated in a simpler way by:

$$y = U_l D_l x + n \qquad (7)$$

where

$$U_l = [U_{1l}, U_{2l}, \ldots, U_{Kl}]$$

$$D_l = diag(D_{1l}, D_{2l}, \ldots, D_{Kl})$$

$$x = [x_1^T, x_2^T, \ldots, x_K^T]$$

Therefore, the post-processing at the BS side can be done using ZF solution $P_s = \left({U_l}^T U_l\right)^{-1} {U_l}^T$.

Then, Eq.7 becomes

$$\breve{x} = D_l x + \breve{n} \qquad (8)$$

Since D_l is exactly diagonal, there is no MUI and the Massive MIMO channel is reduced on to $N \times K$ separate and independent SISO channels. However, the noise component $\breve{n}$ is colored by the post-filter and can be potentially enhanced.

Finally, the transmitted data from all the UEs can be detected by performing a simple equalization process:

$$\breve{x} = (D_l)^{-1} D_l x + (D_l)^{-1} \breve{n} = x + \hat{n} \qquad (9)$$

2.4 Discussion

The SVD Beamforming and ZF MUI cancellation presented in the two previous sub-sections can only be implemented if the UL CSI is known at both the BS and each UE. In the considered FDD system, UEs send pilots sequences which areknown at the BS side. Based on the reception of these pilotssequences, the BS estimates the CSI to each of the UEs. Afterthat, the BS have to feed back the CSI of each UE. There aretwo possibilities:

- •The BS can feed back for each UE $u = 1, \ldots, K$, its FCR matrix H_u. However, this is not optimal for SVD Beamforming scheme. Because in this case, each UE will be forced to perform a SVD decomposition in order toget its precoder matrix.
- •To avoid this previous drawback, the pre-coding matrices which are known at the BS can be directly fed back instead of the FCR.

It is important to note that, in Massive MIMO OFDM system, the BS have to feed back K pre-coding matrices ofsize $N \times N$ for each subcarrier. This feedback overhead can be very important and its reduction is necessary.

3 LOSSLESS FEEDBACK OVERHEAD COMPRESSION

3.1 Givens Rotation Feedback overhead compression

Givens Rotation (GR) is proposed by J. kim and al [19] in order to reduce the feedback overhead exploiting the unitary property of the pre-coding matrix P_{ru}. The GR feedback compression method is adopted in IEEE802.11/ac WLAN standard and its principle is to represent a unitary matrix in aspecial form with complex diagonal matrices as follows:

$$V_u^T = \prod_{i=1}^{\min(\mathrm{R}, N_t - 1)} \left[D_{u,i} \prod_{l=i+1}^{N_t} G_{u,li}^T \left(\psi_{u,li}\right) \right] \times I_{N_t \times R}$$

(10)

Where

$$D_{u,i} = \begin{bmatrix} I_{i-1} & 0 & 0 & \cdots & 0 \\ 0 & e^{j\Phi_{u,ii}} & 0 & \cdots & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & e^{j\Phi_{u,N-1i}} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(11)

$$G_{u,li} = \begin{bmatrix} I_{i-1} & 0 & 0 & \cdots & 0 \\ 0 & \cos(\psi_{u,li}) & 0 & \sin(\psi_{u,li}) & 0 \\ 0 & 0 & I_{l-i-1} & 0 & 0 \\ 0 & -\sin(\psi_{u,li}) & 0 & \cos(\psi_{u,li}) & 0 \\ 0 & 0 & 0 & 0 & I_{N_t - l} \end{bmatrix}$$

(12)

The matrix $D_{u,i}$ is an $N \times N$ diagonal matrix, $I_{N \times N}$ is an $N \times N$ identity matrix, $G_{u,li}$ is an $N \times N$ Givens Rotation matrix. According to the equations (12-13), the parameters to be determined to identify the precoder matrix $P_{ru} = Vu^T$ are:

$\psi_{u,li}$ for $i = 1,2,\dots,N$ and $i < l \le N$
$\Phi_{u,ji}$ for $i = 1,2,\dots,N$ and $i < j \le N$

Therefore, instead of all the elements of the matrix P_{ru}, it is sufficient to consider the parameters $\psi_{u,li}$ and $\Phi_{u,ji}$. Notethat these angles parameters can vary from 0 to 2π for $\Phi_{u,ji}$ and from 0 to π/2 for $\psi_{u,li}$.

Now, $\psi_{u,li}$ and $\Phi_{u,ji}$ can be quantized according to Eq.15 and Eq.16 where $\widehat{\psi}_{u,li}$ and $\widehat{\Phi}_{u,ji}$ represent the quantized angles.

$$\widehat{\psi}_{u,li} = \frac{Q_{\psi}\frac{\pi}{2}}{2^{nb}} + \frac{\pi}{2^{nb+1}} \quad (13)$$

Where $Q_{\psi} = 0,1,2,\dots,2^{nb} - 1$ and nb the number of bits per angle ψ.

$$\widehat{\Phi}_{u,ji} = \frac{Q_{\Phi} 2\pi}{2^{nb}} + \frac{\pi}{2^{nb}} \quad (14)$$

Where $Q_{\Phi} = 0,1,2,\dots,2^{nb} - 1$ and nb the number of bits per angle Φ.

The total number of bits N_{FGR} can be calculated using the formula $N_{FGR} = N_o \times nb \times N(N-1) \times K$, where N_o represents the number of OFDM subcarriers. Based on the fact that the feedback overhead without compression is $N_o \times nb \times N^2 \times K$, the feedback overhead compression rate can be expressed by $(N-1)/2N$. Unfortunately, thisfeedback overhead compression rate decreases with the growth of antenna ports. This last disadvantage limits the performance of GR in Massive MIMO.

However, the GR rotation remains an interesting method because, as mentioned previously, the SVD decomposition is only performed at the BS side. Therefore, to take advantage of this particularity of the GR approach, we propose its improvement in the section below.

3.2 Improvement of the Givens Rotation compression using LZW algorithm

The LZW lossless compression is a simple dictionary coding and its purpose is to replace binary sequences of predetermined length with codes that are their positions in the created dictionary [21]. The LZW encoding and decoding algorithms are respectively presented in Algorithm1 and Algorithm2, where BDC and DBC stand for respectively binary to decimal conversion and decimal to binary conversion.

Algorithme1 : LZW Feedback overhead encoding

```
Initialize the dictionary with all integer values from 0 to $2^{n_b} - 1$
Consider for each input binary sequence of length $n_b$ his BDC as a code.
$V_{max} = 2^{n_b} - 1$
W = first input binary sequence of length $n_b$
While not end of the input $n_b$ sequences do
    C = next input binary sequence of length $n_b$
    If the code of W concatenated with C is in the dictionary then
        Replace W by W concatenated with C
    else
        $V_{max} = V_{max} + 1$
        Output the code of W
        Add $V_{max}$ as the code of W concatenated with C in the dictionary
        Replace W by C
Output the code of W
```

Algorithme 2 : LZW Feedback overhead decoding

```
Initialize the dictionary with all integer values from 0 to $2^{n_b} - 1$
$V_{max} = 2^{n_b} - 1$
OLD = first input code of the codes stream
Output the DBC of OLD
While not end of input codes stream do
    CURRENT = next input code of the codes stream
    Output the sequence of $n_b$ or a multiple of $n_b$ bits corresponding to CURRENT in the dictionary
    $V_{max} = V_{max} + 1$
    Add $V_{max}$ in the dictionary as the code of the sequence of bits corresponding to OLD concatenated with the first $n_b$ bits of that corresponding to CURRENT
    Replace OLD by CURRENT
```

As can be seen in the LZW encoding and decoding:

- the LZW algorithm is very simple. In addition to this, the dictionary is designed at the same time as the encoding or the decoding and does not need to be saved.

•the efficiency of the algorithm increases as the number of repetitive sequences in the input data increases. To improve the GR feedback compression, the LZW algo-rithm is applied, as shown in Fig.2, to the output quantized angles $\widehat{\psi}$ and $\widehat{\Phi}$ which are sequences of lengthnbbits.

•At the BS side, after the GR compression, all the $N_o{\times}N(N-1){\times}K$ output sequences of length nb bits are concatenated to get the LZW input binary sequence.

•At each UE side, the LZW decoding algorithm is used before the GR one in order to retrieve the precoding matrix P_{ru}, where $u = 1, \dots, K$.

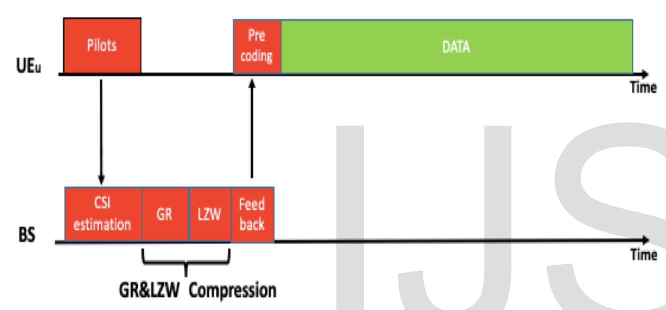

Fig. 2. FDD UL CSI estimation, compression and feedback for one UE.

4 SIMULATION RESULTS

4.1 Simulation Settings

The considered simulation parameters are listed in Table 1. We consider several values of N_o in order to make simulations that concern a large part of modern wireless communication systems including 5G where N_o can be 1024, IEEE802.11 where N_o is up to 512 and IEEE802.16 where $N_o = 256$.

4.2 Validation of the number of quantizer bits per angle

The Figures 3 and 4 show the bit error rate (BER) as a function of signal to noise ratio (SNR) in multi-user UL FDD massive MIMO OFDM context. The modutions are respectively 4-QAM and 16-QAM. Two schemes are considered, one with quantized feedback hoverhead compression by considering several values of nb and one without compression. Several values of the parameters M, N and K are also considered.

As we can see, the quatization error is negligible when nb reaches 8bits. The performances with or without quantizationare the same in all considered configurations. Therefore, $nb = 8$ is enouph for the elimination of the quantization noise affect in the system performance. It is this number of quantizer bit $nb = 8$ that has been adopted in IEEE 802.11ac standard [20].

TABLE 1

SIMULATION PARAMETERS.

Channel model	Rayleigh flat fading
N_0	128,256,512 et 1024
$n_b(bit)$	2,4,6,8 et 10
Cyclic prefix	1/4
M	64 et 128
N	4 et 8
K	16 et 32
Modulation	4-QAM et 16-QAM
FEC	Convolutional
Coding Rate	1/2

4.3 Compression rate

The compression rates of conventional GR and the proposed GR with LZW (GR&LZW) based feedback overhead compression for different Massive MIMO configurations and number of OFDM subcarriers (N_o) are listed in table 2. Note that the number of quantizer bits is fixed to $nb = 8$ as previously validated.

As demonstrated in section 3.1, the compression rate of conventional GR is $(N + 1)/2N$. That is why, in table 2, for any Massive MIMO configuration and any number of OFDM subcarriers N_o, the compression rates are 56.25.5% and 62.5% when the numbers of antennas at each UE are respectively $N = 4$ and $N = 8$.

Unlike the conventional GR compression rate which only depends on the number of antennas per UE, the GR&LZW one takes advantage on the evolution of others parameters, specially the number of OFDM subcarriers. Because, the more N_o increases, the more there is repetitive binary sequences of length nb. Accordingly, the compression rate increases considerably as

TABLE 2

COMPRESSION RATE OF THE CONVENTIONAL GIVENS ROTATION AND THE PROPOSED GIVENS ROTATION WITH LZW ALGORITHMS IN DIFFERENT MASSIVE MIMO OFDM CONFIGURATIONS. M,N,K AND N_o RESPECTIVELY STAND FOR THE NUMBER OF ANTENNAS AT THE BS, THE NUMBER OF ANTENNAS AT EACH UE, THE NUMBER OF UES AND THE NUMBER OF OFDM SUBCARRIERS

Massive MIMO OFDM $\boldsymbol{M \times N \times K \times N_0}$	Feedback amount without compression in bit	Feedback amount with **GR** compression in bit	Compression Rate	Feedback amount with **GR&LZW** compression in bit	Compression Rate
$64 \times 4 \times 16 \times 128$	524288	196608	62.5%	35664	93.20%
$64 \times 4 \times 16 \times 256$	1048576	393216	62.5%	55224	94.73%
$64 \times 4 \times 16 \times 512$	2097152	786432	62.5%	78611	96.25%
$64 \times 4 \times 16 \times 1024$	4194304	1572864	62.5%	120288	97.13%
$128 \times 4 \times 32 \times 128$	1048576	393216	62.5%	77259	92.63%
$128 \times 4 \times 32 \times 256$	2097152	786432	62.5%	118944	94.32%
$128 \times 4 \times 32 \times 512$	4194304	1572862	62.5%	169274	95.96%
$128 \times 4 \times 32 \times 1024$	8388608	3145728	62.5%	257730	96.93%
$64 \times 8 \times 8 \times 128$	1048576	458752	56.25%	90051	91.42%
$64 \times 8 \times 8 \times 256$	2097152	917504	56.25%	138628	93.38%
$64 \times 8 \times 8 \times 512$	4194304	1835008	56.25%	197288	95.29%
$64 \times 8 \times 8 \times 1024$	8388608	3670016	56.25%	300510	96.42%
$128 \times 8 \times 16 \times 128$	2097152	9175504	56.25%	193592	90.77%
$128 \times 8 \times 16 \times 256$	4194304	1835008	56.25%	296805	92.92%
$128 \times 8 \times 16 \times 512$	8388608	3670016	56.25%	422475	94.96%
$128 \times 8 \times 16 \times 1024$	16777216	7430032	56.25%	640704	96.18%

the structure of LZW algorithm enables improvement of compression capacity when the repetitive sequences become large.

Then, observing the behavior of the GR&LZW compression rate in the table 2, we can note that, for the same Massive MIMO configuration (when M, N and K are fixed), the compression rate of the proposed GR&LZW increases with the number of OFDM subcarriers N_o.

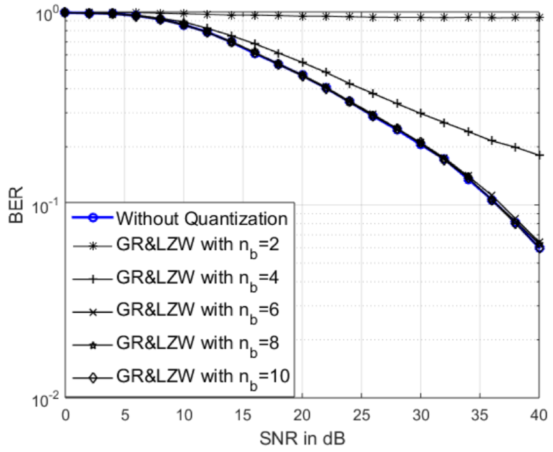

Fig. 3. BER performance in UL FDD Massive MIMO OFDM context $M = 64$, $N = 4$, $K = 16$ and the modulation is 4-QAM.

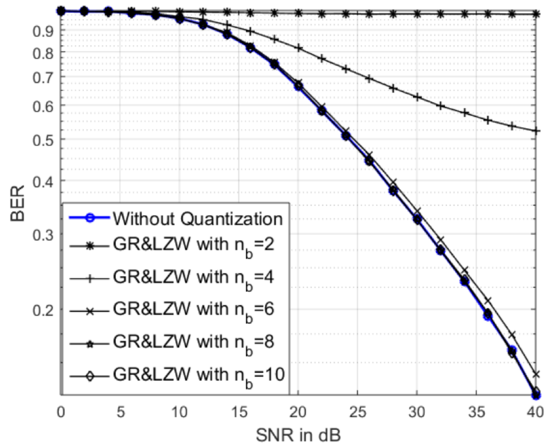

Fig. 4. BER performance in UL FDD Massive MIMO OFDM context $M = 64$,$N = 4$,$K = 16$ and the modulation is 16-QAM.

5 CONCLUSION

Multi-user FDD Massive MIMO OFDM is a promisin scheme for providing unprecedented spectral efficiency. Several powerful pres-processing and post-processing techniques are proposed, in the literature, in order to both built the Beam and eliminate the potential multi-user interference.

However, since both UL and DL CSI are mandatory at each transceiver side, FDD Massive MIMO OFDM systems require a large amount of CSI feedback. Consequently, limited feedback is now an important research topic.

In this paper, we propose an improvement of the con-ventional Givens Rotation feedback compression which was adopted in IEEE 802.11 standard. The proposed feedback compression, maned GR&LZW, is based on LWZ lossless compression. The performances of the proposed GR&LZW are evaluated considering several UL Massive MIMO OFDM configurations. Whatever the system configuration, the GR&LZW out perform the GR one and reaches a compression rate which exceeds 90%.

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